

Okay, so I need to check if the polynomial x^3+4x+9 has any roots. Hmm, let me think. First, I remember that for polynomials, a root is a value of x that makes the polynomial equal to zero. So, I need to find if there's any real number x such that $x^3+4x+9=0$.

Wait, but the question says "check this polynomial for roots"—does it mean real roots or complex roots? Since it's a cubic polynomial, it must have at least one real root by the Fundamental Theorem of Algebra, but maybe they want to know about real roots specifically. Let me assume they mean real roots first.

Let me try to analyze the function $f(x)=x^3+4x+9$. Wait, no, it's $x^3 + 4x + 9$. Let me write that down: $f(x) = x^3 + 4x + 9$. To find real roots, I can look at the graph of this function or use calculus to check for sign changes.

First, let's check the behavior of the function as x approaches positive and negative infinity. As $x \rightarrow \infty$, x^3 dominates, so $f(x) \rightarrow \infty$. As $x \rightarrow -\infty$, x^3 dominates, so $f(x) \rightarrow -\infty$. So the function goes from negative infinity to positive infinity, which means it must cross the x -axis at least once, so there is at least one real root. But maybe the question is about whether there are any rational roots? Let me check using the Rational Root Theorem.

The Rational Root Theorem says that any possible rational root p/q of a polynomial with integer coefficients must have p dividing the constant term and q dividing the leading coefficient. Here, the constant term is 9 and the leading coefficient is 1, so possible rational roots are $\pm 1, \pm 3, \pm 9$.

Let me test these:

$$\text{For } x=-1: (-1)^3+4(-1)+9=-1-4+9=4 \neq 0$$

$$x=1: 1+4+9=14 \neq 0$$

$$x=-3: (-3)^3+4(-3)+9=-27-12+9=-30 \neq 0$$

$$x=3: 27+12+9=48 \neq 0$$

$$x=-9: (-9)^3+4(-9)+9=-729-36+9=-756 \neq 0$$

$$x=9: 729+36+9=774 \neq 0$$

So none of the rational roots work. That means there are no rational roots, but there must be at least one real root. Maybe the question is asking for real roots? Let me check if the function has only one real root or three.

To determine the number of real roots, I can use the derivative. Let's compute $f'(x)=3x^2+4$. The derivative is always positive because $3x^2$ is non-negative, and adding 4 makes it always at least 4. So the function is strictly increasing everywhere. Therefore, it can cross the x -axis only once. So there is exactly one real root, and two complex conjugate roots.

Wait, but the question says "check this polynomial for roots"—maybe they just want to know if there are any roots at all, but since it's a cubic, there must be at least one real root. Let me confirm with the Intermediate Value Theorem. Let's pick some values of x to see where the function changes sign.

$$\text{Let's try } x = -2: (-2)^3+4(-2)+9=-8-8+9=-7$$

$$x = -1: \text{we did that, } f(-1) = 4$$

So between $x = -2$ and $x = -1$, the function goes from -7 to 4 , so by Intermediate Value Theorem, there's a root there.

$x = -3$: $f(-3) = -30$, $x = -2$: -7 , so it's increasing, so only one real root.

So the polynomial has one real root and two complex roots.

But the question is "check this polynomial for roots"—maybe they just want to know if there are any roots, but since it's a cubic, it has roots (in complex numbers). But maybe the user is asking for real roots.

Wait, the original question says "check this polynomial for roots". So the user wants to check if the polynomial x^3+4x+9 has roots (real or complex).

In complex numbers, every non-constant polynomial has roots (Fundamental Theorem of Algebra), so this cubic has three roots in complex numbers. But if they mean real roots, then as we saw, there's exactly one real root.

But let me see if the user might have meant something else. Maybe they want to factor it? But since it has no rational roots, it's irreducible over rationals, but it factors into a linear term and a quadratic over reals.

Let me try to find the real root approximately. Let's use Newton-Raphson method. Let's take $x_0 = -1.5$.

$$f(-1.5) = (-1.5)^3 + 4(-1.5) + 9 = -3.375 - 6 + 9 = -0.375$$

$f(-1) = 4$, so root between -1.5 and -1 .

$f(-1.5) = -0.375$, $f(-1) = 4$. Let's take $x_0 = -1.5$.

$$f'(x) = 3x^2 + 4, \text{ so } f'(-1.5) = 3(2.25) + 4 = 6.75 + 4 = 10.75$$

Next iteration: $x_1 = x_0 - f(x_0)/f'(x_0) = -1.5 - (-0.375)/10.75 \approx -1.5 + 0.035 \approx -1.465$

$$f(-1.465) \approx (-1.465)^3 + 4(-1.465) + 9$$

Calculate $(-1.465)^3$: $1.465^2 = 2.146$, so $1.465^3 \approx 2.146 \cdot 1.465 \approx 3.14$, so $(-1.465)^3 \approx -3.14$

$$4(-1.465) = -5.86$$

So $f(-1.465) \approx -3.14 - 5.86 + 9 = 0$. So approximately -1.465 is a root.

So the polynomial has one real root around -1.465 and two complex roots.

But the question is just to "check this polynomial for roots". So the answer is: the polynomial has one real root and two complex roots.

Wait, but maybe the user is from a context where "roots" refers to real roots. Let me check the original question again: "check this polynom for roots, x^3+4x+9 ". So probably they want to know if there are real roots, or if it has roots at all.

Since it's a cubic, it must have at least one real root, so the answer is yes, it has roots (in fact, one real root).

But let me make sure I didn't make a mistake in the derivative. $f(x) = x^3 + 4x + 9$, so $f'(x) = 3x^2 + 4$, which is always positive, so the function is strictly increasing, so only one real root.

Yes, that's correct. So the polynomial has exactly one real root and two complex conjugate roots.

So the answer is: The polynomial x^3+4x+9 has one real root and two complex roots. Specifically, there is exactly one real root (approximately -1.465) and two complex roots.

But the user might just want a simple check. Let me see.

Alternatively, maybe they made a typo in the polynomial, but assuming it's correct, the answer is as above.

I think that's it.